

Evaluating VaR Models

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February, 2006

Abstract

Financial crisis those we have been experienced during last two decades encouraged the efforts of both academicians and the market participants to develop clear representations of the risk exposure of a financial institute. After introduction of Value-at-Risk by J.P. Morgan, Value-at-Risk has emerged as the standard. However, there are several alternative Value-at-Risk implementations which may produce significantly different Value-at-Risk forecasts. Thus, evaluation of Value-at-Risk forecasts is very crucial. Contributions of the paper focus on three sources. First, two duration based tests are suggested by using the setup defined by Christoffersen and Pelletier[6]. Second, a new loss function is defined to reflect the concerns of the firm. Finally, 13 simulation based Value-at-Risk models are compared by using portfolios which contain fx, stock and bonds.

JEL:C52

Keywords: Value-at-Risk, model evaluation, conditional coverage, duration based coverage testing, loss functions

1 Introduction

Representation of the risk exposure of a financial institute has been a demanding issue for risk managers. Especially, financial crisis those we have been experienced during last two decades encouraged the efforts of both academicians and the market participants. Financial institutions looked for meaningful information about their risk exposure without the need for further technical explanations. In this situation, J.P. Morgan[16] developed Value-at-Risk (VaR) concept that has emerged as the standard.

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VaR is just a single amount that reflects the worst possible loss of an asset portfolio for a given confidence level. In other words, VaR is a quantile of the conditional profit-loss distribution. Although VaR concept intuitively and simply addresses the risk exposure, there is no unique procedure to forecast VaR. There are various VaR implementations those can be classified into two main categories. One approach uses parametric methods and makes a distribution assumption. The other approach simulates the profit-loss distribution and calculates required percentile for this simulated distribution.

Financial risk managers have to select a proper model among the space of possible implementations, because all VaR models do not work well for every markets. The risk of financial risk model is called "model risk"; and is very important phenomenon in risk management. Therefore, evaluation of VaR model among a wide variety of alternative methods is the key element of VaR calculation.

One way to evaluate a VaR model is to employ statistical hypothesis testing methods under the null hypothesis that the model satisfies necessary theoretical conditions. In an early study about model evaluation, Kupiec[19] proposes several tests those are available and finds these tests have very limited power for commonly used sample sizes. Christoffersen[7] underlines the importance of violation clustering and improves testing framework to include conditional coverage. Recently, Christoffersen and Pelletier [6] suggests a new statistical testing framework which is based on duration of violation. They find that these new tests show better power properties with simulated data.

Another way of forecast evaluation is to incorporate a subjective loss functions that reflects the utility maximizing behavior of the financial institution. Lopez[20] formalizes this kind of methods and defines different loss functions for different financial institutions those have different utility functions.

In this paper, two new duration based test are introduced and one method is developed for describing profit maximization of the financial firms. These new statistical tests and loss function are compared with current tests by using 13 simulation based VaR models in two portfolios. Paper also examines model selection problem.

The rest of the paper is outlined as follows: in the following section, model evaluation methods are described. In the next chapter, performances of the simulation based models are compared. Finally, I conclude.

2 Model Evaluation

Let $\tilde{\Phi}$ be the conditional distribution of daily logarithmic returns of a portfolio, R_t , then definition of VaR forecast, v_t^α is;

$$v_t^\alpha = -\tilde{\Phi}^{-1}(\alpha | \mathfrak{S}_{t-1}) \quad (1)$$

We define violation sequence of VaR forecast as;

$$I_t = \begin{cases} 1, & \text{if } (R_t < -v_t^\alpha) \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

A quick theoretical result from these definitions is that for a proper VaR model probability of having a violation should be α . Most statistical evaluation methods exploit this feature of the VaR forecasts.

One important problem for considering violations as an indicator series is that it ignores the magnitude of violation. However the magnitude of violation is very meaningful for the regulatory authorities. At the same time, these techniques do not consider overestimation too. For instance, consider two VaR forecasts v_t^α and $(v_t^\alpha)^+$ where the second one is defined as v_t^α plus a small constant ε which satisfies $\varepsilon < \min(I_t \cdot (R_t + v_t^\alpha))$. Then violation sequence of two VaR forecasts will be identical and any testing procedure based only on $\{I_t\}_{t=1}^T$ will produce same results for both VaR forecasts, however first model is more desirable for the firms and the second is more desirable for the regulators. Thus, beyond statistical tests, incorporating loss functions might be useful. Mandira et al.[21] suggests a two step model selection method which contains a first step of statistical evaluation and a second step of loss functions.

In the empirical results, VaR forecasts will be analyzed by employing 4 statistical tests and 2 loss functions. For each portfolio, paper reports selected model or models by following a two step procedure similar to Mandira et al.

2.1 Unconditional and Conditional Coverage Tests

For a sequence of VaR forecasts that calculated by using a proper model, $\{v_t^\alpha\}_{t=1}^T$, each element of violation sequence, $\{I_t\}_{t=1}^T$, can be modelled as independent draws from a Bernoulli distribution with probability of having a violation is α . Christoffersen[7] suggest a likelihood ratio test for

$$H_0 : \hat{\alpha} = \alpha \quad (3)$$

where $\hat{\alpha}$ ML estimate of α . Likelihood of an i.i.d. Bernoulli distributed sequence can be written as

$$L(\alpha) = \prod_{t=1}^T (1 - \alpha)^{1-I_t} \alpha^{I_t} = (1 - \alpha)^{T_0} \alpha^{T_1} \quad (4)$$

where T_0 is the number of covered days and T_1 is the number of violations. ML estimate of α is

$$\hat{\alpha} = \frac{T_1}{(T_0 + T_1)} = \frac{T_1}{T} \quad (5)$$

Now, we can easily find the likelihood of the sample by plugging the ML estimate into equation 4;

$$L(\hat{\alpha}) = \left(1 - \frac{T_1}{T}\right)^{T_0} \left(\frac{T_1}{T}\right)^{T_1} \quad (6)$$

Then, likelihood ratio test for unconditional coverage is

$$LR_{uc} = 2(l(\hat{\alpha}) - l(\alpha)) \stackrel{asy}{\sim} \chi_1^2 \quad (7)$$

where $l(.)$ is the log-likelihood function which defined as $\ln(L)$. Christoffersen[7] showed that LR_{uc} is asymptotically χ^2 distributed with degrees of freedom 1, however most likely we do not have large samples for VaR evaluation. Therefore, it is better to simulate LR_{uc} for finite samples. In this study, I will use Monte Carlo simulation technique for p-values¹.

Unconditional coverage test implicitly assumes that the violations are independent over time. This assumption ignores clustering of violation which means that violations can occur closely together. If violations are clustered, probability of having a violation after a violation will be higher than α . In order to test existence of such an effect, we can define a first order Markov sequence with transition matrix

$$A = \begin{bmatrix} \alpha_{00} & \alpha_{01} \\ \alpha_{10} & \alpha_{11} \end{bmatrix} \quad (8)$$

where α_{00} is the probability of having a covered day after a covered day, α_{01} is the probability of having a violation after a covered day and so on... With this setup, independence can be defined as the null hypothesis that

$$H_0 : \alpha_{01} = \alpha_{11} \quad (9)$$

¹All p-values are calculated by simulating test statistics 10.000 times.

The likelihood function of first order Markov process can be written as

$$L(A) = \alpha_{00}^{T_{00}} \alpha_{01}^{T_{01}} \alpha_{10}^{T_{10}} \alpha_{11}^{T_{11}} \quad (10)$$

The ML estimates of elements in the transition matrix are

$$\hat{\alpha}_{01} = \frac{T_{01}}{T_{00} + T_{01}} \quad (11)$$

$$\hat{\alpha}_{11} = \frac{T_{11}}{T_{10} + T_{11}} \quad (12)$$

$$\hat{\alpha}_{00} = 1 - \hat{\alpha}_{01} \quad (13)$$

$$\hat{\alpha}_{10} = 1 - \hat{\alpha}_{11} \quad (14)$$

Using ML estimates of parameters, we can calculate likelihood of the sample. Now we can test null hypothesis of independence by using another likelihood ratio test as offered by Christoffersen[7].

$$LR_{ind} = 2 \left(l(\hat{A}) - l(\hat{\alpha}) \right) \overset{asy}{\sim} \chi_1^2 \quad (15)$$

Again test statistic asymptotically a χ^2 with degrees of freedom 1. As final step, test of correct conditional coverage is

$$LR_{cc} = 2 \left(l(\hat{A}) - l(\alpha) \right) = LR_{uc} + LR_{ind} \overset{asy}{\sim} \chi_2^2 \quad (16)$$

which tests $\alpha_{01} = \alpha_{11} = \alpha$. As it is mentioned before, for finite samples, p-values can be calculated from Monte Carlo simulation.

2.2 Duration Based Tests

Although Christoffersen's conditional coverage test provides a parsimonious procedure for model evaluation, it is limited in the sense that it only considers first order dependence. If the violation sequences exhibit a dependence structure other than first order Markov dependence, test would fail to detect.

In a recent paper, Christoffersen and Pelletier[6] suggest a new testing framework which based on duration of violations rather than sequence of violations itself. The motivation behind this approach is that if violations are clustered, there would be relatively short durations and relatively long durations as many as it is unlikely to occur under a proper duration distribution.

No-hit duration between two violations can be simply defined as

$$D_i = t_i - t_{i-1}, \quad t_0 = 0 \quad (17)$$

where t is the day of violation and i is the number of violation. Therefore first duration is equal to day of first violation.

As it is remarked above, each element of violation sequence comes from a Bernoulli distribution. Thus, if we consider duration d , as $(d - 1)$ times consecutive non-violations and one violation at d^{th} trial, probability distribution of no-hit duration is

$$\Pr(D = d) = (1 - \alpha)^{d-1} \alpha \quad (18)$$

which is called geometric distribution. Expected duration for geometrically distributed random variable is $\frac{1}{\alpha}$ and the variance is $\frac{1-\alpha}{\alpha^2}$. Dealing with duration distributions, hazard functions are also important, they identifies the characteristics of the distribution. Hazard function of duration distribution is defined as

$$\lambda(d) = \frac{\Pr(D = d)}{1 - \Pr(D < d)} \quad (19)$$

As a special case geometric distribution has a constant hazard function as follows,

$$\lambda(d) = \frac{(1 - \alpha)^{d-1} \alpha}{1 - \sum_{i=0}^{d-2} (1 - \alpha)^i \alpha} \quad (20)$$

$$\lambda(d) = \alpha \quad (21)$$

A constant hazard function means that duration distribution has no memory. As it will be mentioned later, Christoffersen and Pelletier[6] tests this feature of duration sequence by incorporating Weibull distribution. However, before proceeding through this way, I will propose another duration based test which tests the null hypothesis that duration sequence is from a geometric distribution that has a violation probability equal to α . For this purpose, first it is necessary to define the likelihood function of the geometrically distributed durations. Log-likelihood function which considers censored and uncensored durations can be written as follows;

$$l_{cn} = C_1 \ln(1 - \Pr(D < d)) + (1 - C_1) \ln(\Pr(D = d)) + \sum_{i=2}^{N-1} \ln(\Pr(D = d)) \\ + C_N \ln(1 - \Pr(D < d)) + (1 - C_N) \ln(\Pr(D = d)) \quad (22)$$

where $\{C_i\}_{i=t}^T$ is the sequence of indicators, it shows a duration is censored ($C_i = 1$) or not ($C_i = 0$). Thus, for all durations this indicator will be 0, except first and last durations. If the first element of violation sequence is 1, then $C_1 = 0$, otherwise $C_1 = 1$, meaning first duration is left censored. Similarly, if the last element of violation sequence is 1, then $C_N = 0$, otherwise

$C_N = 1$, which means that last duration is right censored. $(1 - \Pr(D < d))$ is also called as survival function and for geometric distribution it is defined as

$$S(d) = (1 - \alpha)^{d-1} \quad (23)$$

Inserting equation 18 and 23 into 22 and rearranging, we will have

$$l_{cn}(\alpha) = -C_1 \ln(\alpha) - C_N \ln(\alpha) + N \ln(\alpha) + \ln(1 - \alpha) \sum_{i=1}^N (D_i - 1) \quad (24)$$

and ML estimate can be found as

$$\hat{\alpha} = \frac{N - C_1 - C_N}{\left(\sum_{i=1}^N D_i\right) - C_1 - C_N} \quad (25)$$

Now we can test the null hypothesis that claim

$$H_0 : \hat{\alpha} = \alpha \quad (26)$$

by using the following likelihood ratio,

$$LR_{geo} = 2(l_{cn}(\hat{\alpha}) - l_{cn}(\alpha)) \quad (27)$$

For finite sample inference, again we can benefit from the advantages of Monte Carlo techniques. A useful description of the Monte Carlo procedure can be found in Christoffersen and Pelletier[6].

After defining this simple duration based test, let us return back to memory-free nature of the geometric distribution and the test suggested by Christoffersen and Pelletier. First of all, geometric distribution will be substituted with its continues-time limit, exponential distribution. Thus, distribution of no-hit duration under the null now becomes²

$$f_{exp}(D) = \alpha \exp(-\alpha D) \quad (28)$$

²At this point it is possible to replicate the geometric distribution test by having exponential distribution as the null and alternative instead of geometric distribution. Using equation 22, ML estimate might be found as

$$\hat{\alpha} = \frac{N - C_1 - C_N}{\sum_{i=1}^N D_i}$$

The results of this alternative test are unsurprisingly quite similar, since both distributions are the same at the limit.

To be able to test, memory of hazard function, Christoffersen and Pelletier incorporates Weibull distribution as an alternative, because Weibull distribution allows for duration dependence and independence due to parameter choice. Probability density function of Weibull distribution is

$$f_W(D) = a^b b D^{b-1} \exp\left(- (aD)^b\right) \quad (29)$$

And its hazard function can be formalized as

$$\lambda_w(D) = a^b b D^{b-1} \quad (30)$$

An important property of Weibull function is when $b = 1$, the hazard function becomes a constant function and moreover Weibull distribution reduces to exponential distribution. Therefore, the independence of no-hit duration can be tested using following null hypothesis

$$H_0 : b = 1 \quad (31)$$

Log-likelihood function of the durations again follows the general form given in equation 22. However, this time ML estimates of Weibull parameters a and b are needed to be optimized by using a numerical optimization procedure. Fortunately, it is possible to find following relation between a and b by derivating log-likelihood function

$$\hat{a} = \left(\frac{N - C_1 - C_N}{\sum_{i=1}^N D_i^b} \right)^{\frac{1}{b}} \quad (32)$$

Then, optimization problem becomes a univariate unconstraint maximization. When $b = 1$, equation 32 turns to ML estimate of exponential distribution, $\hat{\alpha}$ (see footnote 2). Hence the null hypothesis 31 implicitly says that

$$D_i \sim Exponential(\hat{\alpha}) \quad (33)$$

In this paper, going one step further, null hypothesis is substituted with

$$D_i \sim Exponential(\alpha) \quad (34)$$

where α is the original coverage of VaR forecast. This approach turns hypothesis 31 to simultaneous hypothesis

$$H_0 : b = 1, \alpha = \hat{\alpha} \quad (35)$$

Once again, test statistic consists of a likelihood ratio test and p-values of this statistics are generated using Monte Carlo methods.

In this paper this new test will be called as modified Weibull test and this test can be shown also as

$$LR_{weibull*} = LR_{weibull} + LR_{exp} \quad (36)$$

where $LR_{weibull*}$ is the modified Weibull test statistic, $LR_{weibull}$ is the Weibull test statistic of Christoffersen and Pelletier and LR_{exp} is the exponential distribution test (see footnote 2).

2.3 Loss Functions

Another way of VaR evaluation is formalized by Lopez[20]. This approach uses loss function that reflects the concerns of related financial institution. A loss functions examines the cost of VaR forecast. Thus, the best VaR model for given loss function minimizes the loss.

In this study, two loss functions are used, one reflects concerns of the regulatory authorities and the other reflects the concerns of the firms. Loss function of regulatory authorities addresses the magnitude of the violations and is first proposed by Lopez[20] as follows

$$l_t^r = \begin{cases} (R_t + v_t^\alpha)^2, & \text{if } (R_t < -v_t^\alpha) \\ 0, & \text{otherwise} \end{cases} \quad (37)$$

Loss function of regulatory authorities reflects two concerns, the number of violations and the magnitude of the violations. The magnitude is important, because even one single violation might be harmful for the financial institution if the magnitude is enormous.

For the firms, concerns are quite different than the regulatory authorities. Firms try to maximize their profit and VaR forecast determine their cost related with capital requirement. From firms' point of view, to be able to compare VaR models, one way might be defining a hypothetic optimum VaR series that creates the minimum possible capital requirements. Hypothetic optimum VaR series can be defined as follows

$$\varpi_t^\alpha = \begin{cases} R_t, & \text{if } (R_t < 0) \\ 0, & \text{otherwise} \end{cases} \quad (38)$$

According to Market Risk Amendment of Basel Capital Accord, market risk capital that a bank must hold is

$$MRC_{t+1} = \sqrt{10} \max \left(v_t^\alpha; k_t \frac{1}{60} \sum_{i=0}^{59} v_{t-i}^\alpha \right) \quad (39)$$

where k_t is multiplier which is determined by number of violations during last 252 days.

Since equation 39 determines the capital requirement of a firm, loss function of the firm must be related with MRC_{t+1} not v_t^α itself. Then first we need to define the minimum possible market risk capital by incorporating equation 38

$$\widetilde{MRC}_{t+1} = \sqrt{10} \max \left(\varpi_t^\alpha; \frac{1}{20} \sum_{i=0}^{59} \varpi_{t-i}^\alpha \right) \quad (40)$$

In this equation k_t multiplier is always equal to its minimum, 3. Finally the loss function for the firms

$$l_t^f = \left(MRC_{t+1} - \widetilde{MRC}_{t+1} \right)^2 \quad (41)$$

This approach incorporates the cost minimization behavior of firms by using a hypothetic minimum capital requirement series. With this approach, not only low VaR forecasts but also high VaR forecasts are penalized. Because k multiplier included to the analyses, this loss function implicitly penalizes violations asymmetrically if they increase k .

3 Empirical Results

In this section, I present application results of the VaR evaluation methods to the simulation based VaR models. For this purpose, I employed two market portfolios, one represents Turkish market instruments and the other covers some US market products³.

Firstly, let us investigate details of the portfolios. Turkish portfolio includes 5 instruments; two zero bonds of Turkish Treasury with 117-day and 453-day maturities⁴, two fx positions (USD/TRL and EUR/TRL), and one stock exchange index (ISE100 Index of Turkey). US portfolio also has similar contents. It consists of 5 instruments too; 2-year TBill, 10-year TBill, NASDAQ, S&P 500 and DOW indexes. Both portfolios have homogeneous present value distribution, so each position has 20% weight in its portfolio.

In this study, I worked roughly 500 VaR results⁵ for both portfolios from November 2003 and November 2005. Since each VaR estimation requires past data, the observations start from November 2002. Another point is parameter

³For Turkish portfolio, all calculations are made in terms of YTL (Yeni Turkish Lira) and all instruments YTL denominated, while it is USD for US portfolio.

⁴Their ISIN codes are TRB220206T14 and TRT240107T12, respectively.

⁵Exact numbers are 505 for Turkish portfolio and 502 for US portfolio.

estimation of volatility and correlation models -if they are required- are done repeatedly. Thus, GARCH(1,1) parameters or DCC(1,1) model parameters are estimated by using a 252-day length moving window of observations.

3.1 Results of the Statistical Tests

In this section, results of the statistical tests are presented. Statistical tests are applied to both 95% and 99% VaR estimations.

3.1.1 Results of 99% VaR

Table 1 and 2 show the results of the unconditional coverage, independence, and conditional coverage tests for 99% VaR forecasts of Turkish portfolio and US portfolio, respectively. First column gives the names of the VaR models. Following three columns provide LR statistics of the tests. Although distribution of these tests are known, as it mentioned before, I preferred applying Monte Carlo method for finite sample inference. Probabilities of LR statistics are given below the LR statistics. Next column gives estimated unconditional coverage probability. And the last two columns shows estimated conditional coverage probabilities.

For Turkish portfolio, any method is rejected. However, for US portfolio, only 4 VaR models are survived (FHS-GARCH, FHS-EWMA-Kernel, WHS, and MC-CCC), and others are rejected at 95% or 90% significance level. This strong divergence of the results is an evidence of differentiation of the proper VaR model from market to market or more specifically from portfolio to portfolio. Thus, risk manager should look for proper models for the portfolio. Another point is all rejection decisions of the tests are the same for all of the models except MC-EWMA of US portfolio. For this case unconditional coverage reject the model but conditional coverage is slightly failed to reject. It is also interesting to notice that all HS variants of US portfolio are rejected.

Now let us examine higher order dependence by incorporating duration based tests. First, I start with geometric distribution test. Table 3 and 4 gives the results of the geometric distribution test. In the first column of the table, method names are given. Next column shows the test statistics and the last gives the coverage probability of the geometric distribution.

Table 1: Results of the unconditional coverage, independence and conditional coverage tests for 99% VaR estimation of Turkish portfolio.

METHODS	LR_{uc}	LR_{ind}	LR_{cc}	$\hat{\alpha}$	$\hat{\alpha}_{01}$	$\hat{\alpha}_{11}$
HS	0.001 (0.818)	0.100	0.101 (0.827)	0.010	0.010	-
HS-EVT	0.237 (0.507)	0.060	0.297 (0.659)	0.008	0.008	-
HS-Kernel	0.984 (0.257)	0.028	1.011 (0.286)	0.006	0.006	-
FHS-EWMA	0.001 (0.818)	0.100	0.101 (0.827)	0.010	0.010	-
FHS-GARCH	0.237 (0.507)	0.060	0.297 (0.659)	0.008	0.008	-
FHS-EWMA-EVT	0.237 (0.507)	0.060	0.297 (0.659)	0.008	0.008	-
FHS-GARCH-EVT	0.170 (0.677)	0.148	0.319 (0.528)	0.012	0.012	-
FHS-EWMA-Kernel	0.001 (0.818)	0.100	0.101 (0.827)	0.010	0.010	-
FHS-GARCH-Kernel	0.984 (0.257)	0.028	1.011 (0.286)	0.006	0.006	-
WHS	0.237 (0.507)	0.060	0.297 (0.659)	0.008	0.008	-
MC-EWMA	0.001 (0.818)	0.100	0.101 (0.827)	0.010	0.010	-
MC-CCC	2.414 (0.106)	0.004	2.417 (0.143)	0.004	0.004	-
MC-DCC	2.414 (0.106)	0.004	2.417 (0.143)	0.004	0.004	-

Table 2: Results of the unconditional coverage, independence and conditional coverage tests for 99% VaR estimation of US portfolio.

METHODS	LR_{uc}	LR_{ind}	LR_{cc}	$\hat{\alpha}$	$\hat{\alpha}_{01}$	$\hat{\alpha}_{11}$
HS	10.091** (0.001)	-	10.091** (0.002)	-	-	-
HS-EVT	10.091** (0.001)	-	10.091** (0.002)	-	-	-
HS-Kernel	10.091** (0.001)	-	10.091** (0.002)	-	-	-
FHS-EWMA	8.908** (0.008)	0.725	9.633** (0.010)	0.026	0.027	-
FHS-GARCH	1.514 (0.190)	0.272	1.786 (0.225)	0.016	0.016	-
FHS-EWMA-EVT	3.873* (0.052)	0.428	4.301* (0.075)	0.020	0.020	-
FHS-GARCH-EVT	4.846** (0.021)	0.004	4.850** (0.032)	0.002	0.002	-
FHS-EWMA-Kernel	2.377 (0.106)	0.004	2.381 (0.142)	0.004	0.004	-
FHS-GARCH-Kernel	4.846** (0.021)	0.004	4.850** (0.032)	0.002	0.002	-
WHS	2.377 (0.106)	0.004	2.381 (0.142)	0.004	0.004	-
MC-EWMA	2.580* (0.071)	0.345	2.926 (0.111)	0.018	0.018	-
MC-CCC	2.377 (0.106)	0.004	2.381 (0.142)	0.004	0.004	-
MC-DCC	4.846** (0.021)	0.004	4.850** (0.032)	0.002	0.002	-

Table 3: Results of the geometric distribution test for 99% VaR estimation of Turkish portfolio.

METHODS	LR_{geo}	$\hat{\alpha}$
HS	0.229 (0.618)	0.008
HS-EVT	0.967 (0.470)	0.006
HS-Kernel	2.389 (0.182)	0.004
FHS-EWMA	0.229 (0.579)	0.008
FHS-GARCH	0.967 (0.337)	0.006
FHS-EWMA-EVT	0.967 (0.337)	0.006
FHS-GARCH-EVT	0.000 (0.936)	0.010
FHS-EWMA-Kernel	0.229 (0.618)	0.008
FHS-GARCH-Kernel	2.389 (0.248)	0.004
WHS	0.967 (0.470)	0.006
MC-EWMA	0.229 (0.618)	0.008
MC-CCC	4.862** (0.049)	0.002
MC-DCC	4.862** (0.049)	0.002

Table 4: Results of the geometric distribution test for 99% VaR estimation of US portfolio.

METHODS	LR_{geo}	$\hat{\alpha}$
HS	20.141** (0.000)	-
HS-EVT	20.141** (0.000)	-
HS-Kernel	20.141** (0.000)	-
FHS-EWMA	7.111** (0.039)	0.024
FHS-GARCH	0.719 (0.517)	0.014
FHS-EWMA-EVT	2.613 (0.136)	0.018
FHS-GARCH-EVT	10.050** (0.025)	-
FHS-EWMA-Kernel	4.813** (0.048)	0.002
FHS-GARCH-Kernel	10.050** (0.025)	-
WHS	4.813** (0.082)	0.002
MC-EWMA	1.538 (0.297)	0.016
MC-CCC	4.813** (0.048)	0.002
MC-DCC	10.050** (0.025)	-

Geometric distribution test rejects MC-CCC and MC-DCC forecasts of Turkish portfolio. In the estimation of CCC and DCC models, GARCH(1,1) model is used as univariate volatility specification and normality is assumed. Then we can say, conditional correlation models that employs GARCH(1,1) with normality is not capable of reflecting correlation structure of Turkish markets for the analysis period, because MC-EWMA is survived with a high probability although it has the same features with MC-CCC and MC-DCC except covariance modelling. For US portfolio, 3 models are failed to reject (FHS-GARCH, FHS-EWMA-EVT, and MC-EWMA), other models are rejected.

Table 5 and 6 present the results of the modified Weibull, Weibull, and exponential distribution tests for 99% VaR forecasts of Turkish portfolio and US portfolio, respectively. Again, first column gives the names of the VaR models and next three columns provide LR statistics of the tests. Following column gives estimated coverage probability for exponential distribution. And the last two columns shows estimated a and b parameters of Weibull distribution.

Test statistics of the exponential distribution is quite similar. However, I observed that distribution of test statistics differs. Distribution of geometric test statistic has a longer right tail while distribution of exponential test statistic is flatter at the center of distribution. For Turkish portfolio, exponential distribution test rejects MC-CCC and MC-DCC models as they rejected by the geometric distribution test, however exponential distribution test rejects at 90% significance while the geometric distribution test rejects at 95% significance. For US portfolio, 3 models are failed to reject (FHS-GARCH, FHS-EWMA-EVT, and MC-EWMA), other models are rejected by at least one of the tests. Another example for the difference between geometric distribution test and exponential distribution test is WHS forecast of US portfolio; for this case geometric distribution test rejects the null, however exponential is failed to reject. Weibull test and modified Weibull test produces totally different results as they supposed to; Weibull test rejects only WHS forecast, however modified Weibull test rejected 7 models. The reason for difference is, Weibull test deals with the dependence between violations, on the other hand modified Weibull test consider coverage too. A final remark is that when a estimate is zero there is no optimal b , thus solution of b is set of real numbers.

Table 5: Results of the exponential distribution, Weibull and modified Weibull tests for 99% VaR estimation of Turkish portfolio.

METHODS	LR_{exp}	$LR_{weibull}$	$LR_{weibull^*}$	$\hat{\alpha}$	\hat{a}	\hat{b}
HS	0.235 (0.643)	0.097 (0.759)	0.333 (0.876)	0.008	0.008	1.141
HS-EVT	0.975 (0.356)	1.885 (0.251)	2.860 (0.359)	0.006	0.006	2.048
HS-Kernel	2.395 (0.200)	2.429 (0.198)	4.824 (0.216)	0.004	0.005	2.773
FHS-EWMA	0.235 (0.643)	0.702 (0.467)	0.937 (0.730)	0.008	0.008	1.387
FHS-GARCH	0.975 (0.442)	1.647 (0.284)	2.623 (0.397)	0.006	0.006	1.867
FHS-EWMA-EVT	0.975 (0.442)	1.647 (0.284)	2.623 (0.397)	0.006	0.006	1.867
FHS-GARCH-EVT	0.000 (0.942)	0.533 (0.521)	0.534 (0.822)	0.010	0.010	1.276
FHS-EWMA-Kernel	0.235 (0.643)	0.226 (0.656)	0.461 (0.838)	0.008	0.008	1.206
FHS-GARCH-Kernel	2.395 (0.157)	0.006 (0.911)	2.401 (0.457)	0.004	0.004	0.959
WHS	0.975 (0.356)	1.885 (0.251)	2.860 (0.359)	0.006	0.006	2.048
MC-EWMA	0.235 (0.584)	1.747 (0.270)	1.982 (0.506)	0.008	0.008	1.671
MC-CCC	4.861* (0.083)	0.149 (0.713)	5.010 (0.194)	0.002	0.001	0.729
MC-DCC	4.861* (0.083)	0.149 (0.713)	5.010 (0.194)	0.002	0.001	0.729

Table 6: Results of the exponential distribution, Weibull and modified Weibull tests for 99% VaR estimation of US portfolio.

METHODS	LR_{exp}	$LR_{weibull}$	$LR_{weibull*}$	$\hat{\alpha}$	\hat{a}	\hat{b}
HS	20.080** (0.000)	- (0.962)	20.080** (0.001)	-	-	{ \mathbb{R} }
HS-EVT	20.080** (0.000)	- (0.962)	20.080** (0.001)	-	-	{ \mathbb{R} }
HS-Kernel	20.080** (0.000)	- (0.962)	20.080** (0.001)	-	-	{ \mathbb{R} }
FHS-EWMA	6.955** (0.039)	0.567 (0.514)	7.522 (0.112)	0.024	0.023	1.197
FHS-GARCH	0.695 (0.536)	3.627 (0.133)	4.322 (0.249)	0.014	0.014	1.965
FHS-EWMA-EVT	2.548 (0.143)	2.243 (0.225)	4.791 (0.228)	0.018	0.017	1.553
FHS-GARCH-EVT	10.040** (0.008)	- (0.962)	10.040** (0.056)	-	-	{ \mathbb{R} }
FHS-EWMA-Kernel	4.813** (0.044)	0.678 (0.479)	5.491 (0.169)	0.002	0.001	0.544
FHS-GARCH-Kernel	10.040** (0.008)	- (0.962)	10.040** (0.056)	-	-	{ \mathbb{R} }
WHS	4.813 (0.122)	9.496** (0.037)	14.309** (0.036)	0.002	0.003	81.801
MC-EWMA	1.496 (0.306)	0.211 (0.676)	1.707 (0.546)	0.016	0.016	1.138
MC-CCC	4.813** (0.044)	0.678 (0.479)	5.491 (0.169)	0.002	0.001	0.544
MC-DCC	10.040** (0.008)	- (0.962)	10.040** (0.056)	-	-	{ \mathbb{R} }

3.1.2 Results of 95% VaR

In this section, evaluation test results of 95% VaR forecasts are analyzed. Table 7 and 8 show the results of the unconditional coverage, independence, and conditional coverage tests for 95% VaR forecasts of Turkish portfolio and US portfolio, respectively. First column gives the names of the VaR models. Following three columns provide LR statistics of the tests. Probabilities of LR statistics are given below the LR statistics. Next column gives estimated unconditional coverage probability. And the last two columns shows estimated conditional coverage probabilities.

Table 7: Results of the unconditional coverage, independence and conditional coverage tests for 95% VaR estimation of Turkish portfolio.

METHODS	LR _{uc}	LR _{ind}	LR _{cc}	$\hat{\alpha}$	$\hat{\alpha}_{01}$	$\hat{\alpha}_{11}$
HS	0.459 (0.476)	2.005	2.464 (0.387)	0.044	0.046	-
HS-EVT	0.797 (0.354)	1.823	2.620 (0.311)	0.042	0.043	-
HS-Kernel	5.094** (0.019)	0.919	6.013** (0.041)	0.030	0.031	-
FHS-EWMA	0.023 (0.843)	0.320	0.343 (0.815)	0.051	0.050	0.077
FHS-GARCH	0.023 (0.843)	0.320	0.343 (0.815)	0.051	0.050	0.077
FHS-EWMA-EVT	0.217 (0.610)	0.002	0.220 (0.878)	0.046	0.046	0.043
FHS-GARCH-EVT	0.561 (0.416)	0.344	0.904 (0.659)	0.057	0.059	0.034
FHS-EWMA-Kernel	0.561 (0.416)	0.344	0.904 (0.659)	0.057	0.059	0.034
FHS-GARCH-Kernel	0.125 (0.688)	0.169	0.294 (0.850)	0.053	0.055	0.037
WHS	0.561 (0.416)	3.536	4.096 (0.120)	0.057	0.061	-
MC-EWMA	0.459 (0.476)	0.002	0.461 (0.754)	0.044	0.044	0.045
MC-CCC	1.774 (0.152)	1.486	3.260 (0.203)	0.038	0.039	-
MC-DCC	1.774 (0.152)	1.486	3.260 (0.203)	0.038	0.039	-

Table 8: Results of the unconditional coverage, independence and conditional coverage tests for 95% VaR estimation of US portfolio.

METHODS	LR_{uc}	LR_{ind}	LR_{cc}	$\hat{\alpha}$	$\hat{\alpha}_{01}$	$\hat{\alpha}_{11}$
HS	0.420 (0.472)	0.001	0.421 (0.789)	0.044	0.044	0.045
HS-EVT	0.745 (0.353)	1.834	2.579 (0.331)	0.042	0.044	-
HS-Kernel	3.089* (0.060)	1.192	4.281 (0.101)	0.034	0.035	-
FHS-EWMA	3.68** (0.049)	0.992	4.672* (0.084)	0.070	0.067	0.114
FHS-GARCH	2.392* (0.099)	0.015	2.407 (0.431)	0.066	0.066	0.061
FHS-EWMA-EVT	0.609 (0.417)	0.353	0.962 (0.640)	0.058	0.059	0.034
FHS-GARCH-EVT	0.000 (0.922)	0.057	0.057 (0.974)	0.050	0.050	0.040
FHS-EWMA-Kernel	0.190 (0.608)	0.003	0.193 (0.883)	0.046	0.046	0.043
FHS-GARCH-Kernel	1.169 (0.262)	0.052	1.221 (0.609)	0.040	0.040	0.050
WHS	0.051 (0.762)	0.022	0.073 (0.942)	0.048	0.048	0.042
MC-EWMA	3.005 (0.083)	0.225	3.229 (0.207)	0.068	0.066	0.088
MC-CCC	0.000 (0.922)	0.057	0.057 (0.974)	0.050	0.050	0.040
MC-DCC	0.190 (0.608)	0.003	0.193 (0.883)	0.046	0.046	0.043

For Turkish portfolio, only HS-Kernel model is rejected. The results of the US portfolio are much more different than the results of 99% VaR forecasts. It is also interesting to notice that FHS-GARCH model which is one of the 4 surviving models among 99% VaR forecasts, is rejected at 95%. HS-Kernel and FHS-EWMA models are rejected at both coverage levels.

Table 9 and 10 gives the results of the geometric distribution test. In the first column of the table, method names are given. Next column shows the test statistics and the last gives the coverage probability of the geometric distribution.

Table 9: Results of the geometric distribution test for 95% VaR estimation of US portfolio.

METHODS	LR_{geo}	$\hat{\alpha}$
HS	0.762 (0.393)	0.042
HS-EVT	1.190 (0.315)	0.040
HS-Kernel	6.156** (0.016)	0.028
FHS-EWMA	0.001 (0.942)	0.050
FHS-GARCH	0.001 (0.942)	0.050
FHS-EWMA-EVT	0.433 (0.507)	0.044
FHS-GARCH-EVT	0.328 (0.617)	0.056
FHS-EWMA-Kernel	0.328 (0.617)	0.056
FHS-GARCH-Kernel	0.030 (0.907)	0.052
WHS	0.328 (0.607)	0.056
MC-EWMA	0.762 (0.393)	0.042
MC-CCC	2.365 (0.117)	0.036
MC-DCC	2.365 (0.128)	0.036

Table 10: Results of the geometric distribution test for 95% VaR estimation of US portfolio.

METHODS	LR_{geo}	$\hat{\alpha}$
HS	0.711 (0.408)	0.042
HS-EVT	1.127 (0.313)	0.040
HS-Kernel	3.888** (0.044)	0.032
FHS-EWMA	3.081* (0.073)	0.068
FHS-GARCH	1.903 (0.174)	0.064
FHS-EWMA-EVT	0.365 (0.577)	0.056
FHS-GARCH-EVT	0.043 (0.778)	0.048
FHS-EWMA-Kernel	0.394 (0.492)	0.044
FHS-GARCH-Kernel	1.647 (0.199)	0.038
WHS	0.173 (0.655)	0.046
MC-EWMA	2.459 (0.116)	0.066
MC-CCC	0.043 (0.791)	0.048
MC-DCC	0.394 (0.492)	0.044

Geometric distribution test rejects HS-Kernel forecast of Turkish portfolio. For US portfolio, again HS-Kernel and FHS-EWMA models are rejected, geometric distribution test reject these models at 99% coverage too.

Table 11 and 12 present the results of the modified Weibull, Weibull, and exponential distribution tests for 95% VaR forecasts of Turkish portfolio and US portfolio, respectively. Again, first column gives the names of the VaR models and next three columns provide LR statistics of the tests. Following column gives estimated coverage probability for exponential distribution. And the last two columns shows estimated a and b parameters of Weibull distribution.

For Turkish portfolio, all HS variants are rejected by at least one of the three tests. All tests rejects HS-Kernel. For US portfolio, 4 models are rejected (HS-Kernel, FHS-EWMA-EVT, FHS-EWMA-Kernel, and MC-EWMA) by at least one of the three tests. Among 4 EWMA related forecasts, 3 models are rejected. Then, it might be an evidence to claim that EWMA model is not a proper model for volatility modelling of US markets.

3.2 Loss Function Results

In this section, VaR methods are compared by employing loss functions which are described before. For loss function evaluation, I will report only results of 99% VaR forecasts, because loss functions are related with the current regulations and regulations require 99% VaR.

Table 13 present the results of the loss functions for Turkish portfolio. First column shows the rank and the second column gives the method and the last provides the loss functions value⁶.

From firms' point of view the best model is FHS-GARCH for the Turkish portfolio. First six models are all FHS variants. It appears that FHS variants are more desirable for the firms. Similarly, last 3 models are HS variants and HS-Kernel is the worst, then it can be said that HS variants are worse performing models from firms' utility maximization perspective. From regulator's point of view, on the other hand, the picture seems to be the opposite. For the regulator, all six FHS variants are worst perform and HS-Kernel is the best model. Monte Carlo models also produce desirable results for the regulator.

⁶FLF and RLF stand for firms' loss function and regulator's loss function, respectively.

FLF is defined as $\sqrt{\frac{\sum_{i=0}^n l_i^f}{n}}$, while RLF is the just sum of l_i^r 's.

Table 11: Results of the exponential distribution, Weibull and modified Weibull tests for 95% VaR estimation of Turkish portfolio.

METHODS	LR_{exp}	$LR_{weibull}$	$LR_{weibull^*}$	$\hat{\alpha}$	\hat{a}	\hat{b}
HS	0.759 (0.373)	3.740* (0.082)	4.499 (0.137)	0.042	0.050	0.761
HS-EVT	1.176 (0.273)	5.161** (0.040)	6.337* (0.062)	0.040	0.050	0.724
HS-Kernel	5.986** (0.015)	4.090* (0.068)	10.077** (0.011)	0.028	0.034	0.695
FHS-EWMA	0.002 (0.949)	0.000 (0.993)	0.003 (0.999)	0.050	0.049	1.002
FHS-GARCH	0.002 (0.949)	0.000 (0.993)	0.003 (0.999)	0.050	0.049	1.002
FHS-EWMA-EVT	0.438 (0.498)	0.130 (0.754)	0.568 (0.786)	0.044	0.045	0.942
FHS-GARCH-EVT	0.289 (0.599)	0.005 (0.949)	0.294 (0.883)	0.055	0.055	1.010
FHS-EWMA-Kernel	0.289 (0.599)	0.005 (0.949)	0.294 (0.883)	0.055	0.055	1.010
FHS-GARCH-Kernel	0.022 (0.894)	0.019 (0.904)	0.041 (0.982)	0.051	0.051	1.021
WHS	0.289 (0.585)	1.394 (0.296)	1.683 (0.497)	0.055	0.061	0.864
MC-EWMA	0.759 (0.415)	0.281 (0.643)	1.040 (0.652)	0.042	0.040	1.097
MC-CCC	2.316 (0.112)	0.642 (0.486)	2.958 (0.276)	0.036	0.034	1.157
MC-DCC	2.316 (0.111)	0.671 (0.475)	2.987 (0.273)	0.036	0.034	1.157

Table 12: Results of the exponential distribution, Weibull and modified Weibull tests for 95% VaR estimation of US portfolio.

METHODS	LR_{exp}	$LR_{weibull}$	$LR_{weibull^*}$	$\hat{\alpha}$	\hat{a}	\hat{b}
HS	0.709 (0.394)	0.892 (0.408)	1.601 (0.506)	0.042	0.040	1.187
HS-EVT	1.115 (0.287)	2.419 (0.165)	3.534 (0.212)	0.040	0.037	1.338
HS-Kernel	3.791** (0.045)	0.934 (0.396)	4.725 (0.124)	0.032	0.030	1.222
FHS-EWMA	2.838 (0.107)	1.774 (0.235)	4.611 (0.130)	0.068	0.064	1.216
FHS-GARCH	1.744 (0.191)	4.214* (0.064)	5.958* (0.070)	0.064	0.059	1.372
FHS-EWMA-EVT	0.323 (0.598)	4.830** (0.048)	5.153* (0.099)	0.056	0.052	1.444
FHS-GARCH-EVT	0.049 (0.841)	1.001 (0.379)	1.050 (0.639)	0.048	0.046	1.179
FHS-EWMA-Kernel	0.400 (0.560)	0.818 (0.428)	1.218 (0.589)	0.044	0.042	1.171
FHS-GARCH-Kernel	1.620 (0.223)	0.075 (0.812)	1.694 (0.480)	0.038	0.037	1.051
WHS	0.181 (0.636)	0.474 (0.550)	0.655 (0.759)	0.046	0.044	1.122
MC-EWMA	2.260 (0.115)	2.917 (0.126)	5.177* (0.098)	0.066	0.062	1.296
MC-CCC	0.049 (0.835)	0.621 (0.493)	0.670 (0.755)	0.048	0.046	1.139
MC-DCC	0.400 (0.541)	1.509 (0.275)	1.909 (0.438)	0.044	0.042	1.241

Table 14 follows the same organization with table 13 and shows results of the loss functions for US portfolio.

In general, table 14 shows that ranks of the models are similar across portfolios. For the firms, FHS variants are, again, at the top of the list while HS variants are at the bottom. Only WHS model exhibits a certain difference, and other models are at similar ranks. For the regulator, the picture is opaque, because 9 models have no violation during the analysis period. However half of the FHS models are at the bottom of the list again.

The results show that loss function approach produce more consistent results across portfolio. Either firms' loss function or regulator's loss function creates quite similar ordering across portfolio. Another result is generally what is good for the firm is bad for the regulator. Firms' loss function suggested by this paper reflected a conflict between the interest of the firms and the regulator.

3.3 Model Selection

In this section, I will apply a model selection methodology which is similar to Mandira et al.[21]. This model selection approach combines statistical approach and loss function approach.

Up to now, we have analyzed 13 VaR models by using both statistical approach and loss function approach. In the model selection phase, I will use a 2 stage approach. First stage is consist of modified Weibull test and the second is the loss function. Thus, for each portfolio, this method will suggest 2 VaR model, one for firms and one for the regulator.

For Turkish portfolio, all of the models are survived in the first stage, modified Weibull test rejected no model. In the second stage, we will use the results of the loss function. Because no model is rejected at the first stage, simply the model that minimizes the loss function, will be selected. Therefore, for the firms FHS-GARCH should be selected and the regulator should select HS-Kernel. For the US portfolio, six models are survived at the first stage, these are; FHS-EWMA, FHS-EWMA-EVT, FHS-EWMA-Kernel, FHS-GARCH-EVT, MC-EWMA and MC-CCC. Then, the model which minimizes the loss function among this six should be the selected model. For the firms this model is FHS-EWMA-EVT. Accordingly for the regulator, selected model is either FHS-GARCH-EVT or MC-CCC.

Table 13: Results of the loss functions for 99% VaR estimation of Turkish portfolio.

RANK	METHOD	FLF	RANK	METHOD	RLF
1	FHS-GARCH	2,560	1	HS-Kernel	215,700
2	FHS-EWMA	2,602	2	MC-CCC	249,927
3	FHS-GARCH-EVT	2,654	3	MC-DCC	257,468
4	FHS-EWMA-EVT	2,728	4	HS-EVT	311,236
5	FHS-GARCH-Kernel	2,881	5	HS	351,759
6	FHS-EWMA-Kernel	2,981	6	WHS	418,886
7	MC-EWMA	3,268	7	MC-EWMA	423,501
8	WHS	3,470	8	FHS-EWMA-Kernel	709,643
9	MC-DCC	3,480	9	FHS-GARCH-Kernel	746,173
10	MC-CCC	3,492	10	FHS-EWMA-EVT	1,027,420
11	HS	3,509	11	FHS-GARCH-EVT	1,042,167
12	HS-EVT	3,598	12	FHS-GARCH	1,161,554
13	HS-Kernel	3,837	13	FHS-EWMA	1,231,083

Table 14: Results of the loss functions for 99% VaR estimation of US portfolio.

RANK	METHODS	FLF	RANK	METHODS	RLF
1	FHS-EWMA-EVT	1,200	1	HS	-
2	WHS	1,269	2	HS-EVT	-
3	FHS-GARCH	1,335	3	HS-Kernel	-
4	FHS-EWMA-Kernel	1,342	4	FHS-GARCH	-
5	FHS-GARCH-EVT	1,357	5	FHS-GARCH-EVT	-
6	FHS-GARCH-Kernel	1,360	6	FHS-GARCH-Kernel	-
7	FHS-EWMA	1,368	7	WHS	-
8	MC-DCC	1,384	8	MC-CCC	-
9	MC-CCC	1,385	9	MC-DCC	-
10	HS	1,389	10	MC-EWMA	217
11	HS-EVT	1,401	11	FHS-EWMA-Kernel	1,550
12	HS-Kernel	1,404	12	FHS-EWMA-EVT	3,591
13	MC-EWMA	1,417	13	FHS-EWMA	8,602

4 Conclusion

This paper have suggested two new tests for model evaluation and proposes a new loss function which mimics the profit maximizing instinct. To investigate performances of these new tests and the loss function, I applied 13 simulation based VaR models to two portfolios which contain fx, bond, and stock positions.

The new statistical tests use the setup that described by Christoffersen and Pelletier [6]. I modified their test to get duration based analogs of unconditional coverage, conditional coverage and independence tests. Empirical results showed that modified version of Weibull test get enabled to detect coverage problem too. For all of the p-values, Monte Carlo analysis are used.

The new loss function is based on a hypothetical VaR measure that produces least possible capital requirement for the firm. In the application, this loss function managed to capture inconsistency of interests between firm and the regulator.

After applying both statistical tests and loss function approach for the model evaluation, I combined them, and proceeded to model selection by employing a similar methodology as it is explained in the Mandira et al[21]. As a natural of this finding optimal models for the firms and the regulator are totally different.

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Appendix- Graphs

Figure 1: 99% VaR forecasts of Turkish portfolio with HS variants

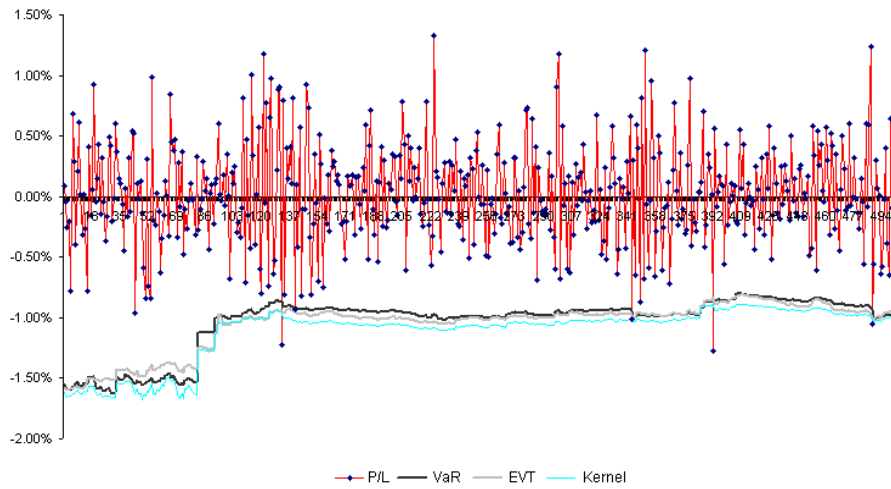


Figure 2: 99% VaR forecasts of Turkish portfolio with FHS-EWMA variants

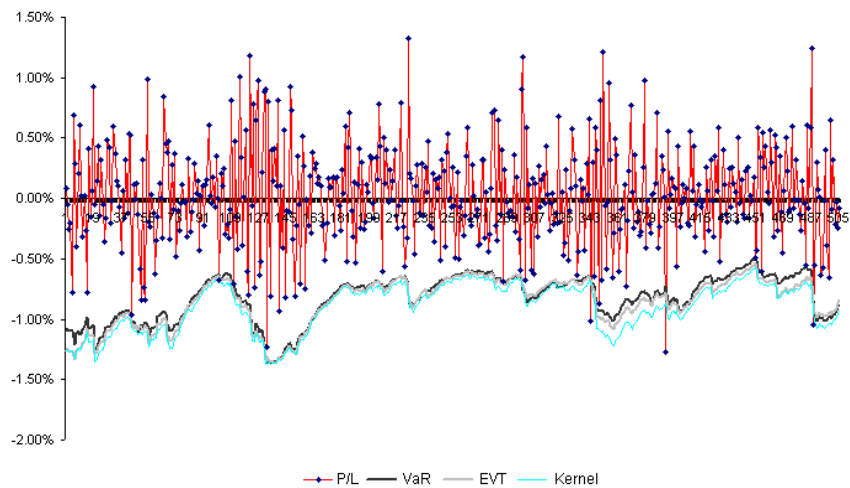


Figure 3: 99% VaR forecasts of Turkish portfolio with FHS-GARCH variants

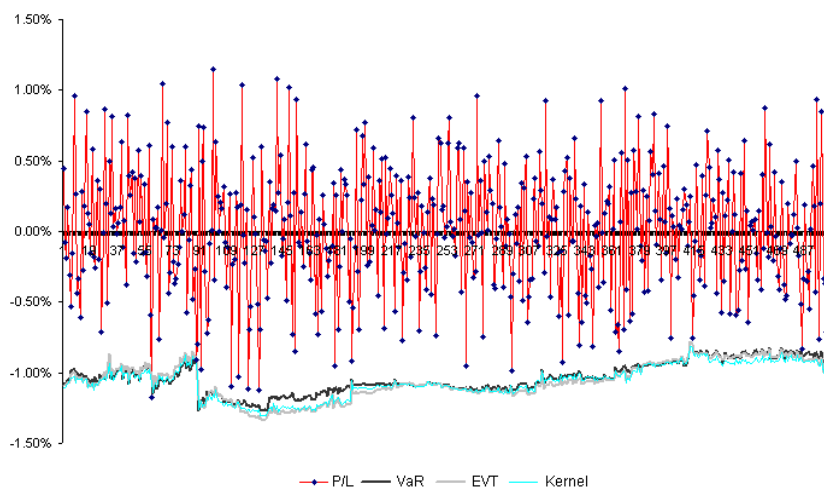


Figure 4: 99% VaR forecasts of Turkish portfolio with WHS

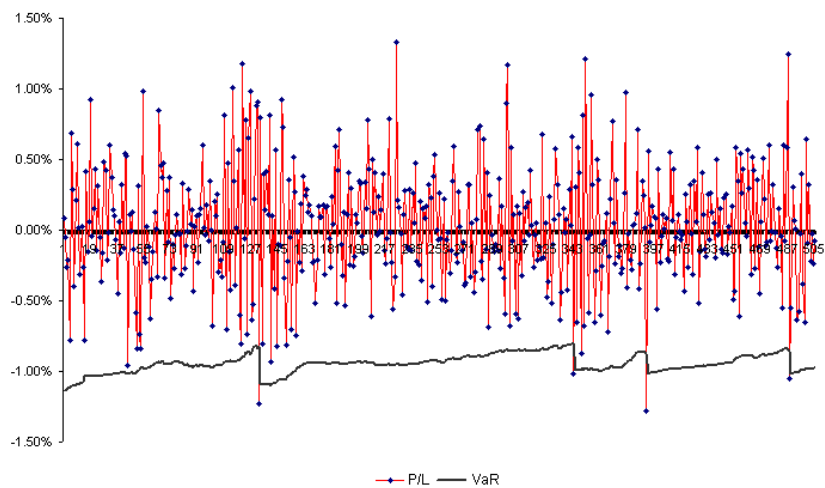


Figure 5: 99% VaR forecasts of Turkish portfolio with MC-EWMA

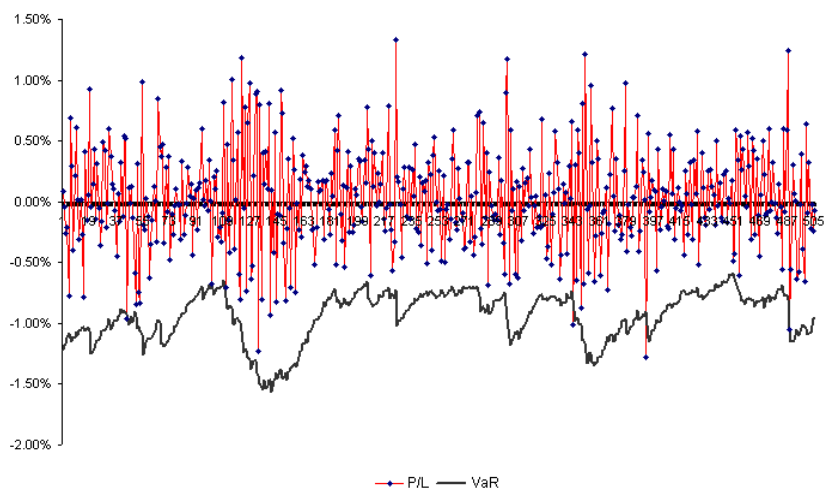


Figure 6: 99% VaR forecasts of Turkish portfolio with MC-CCC

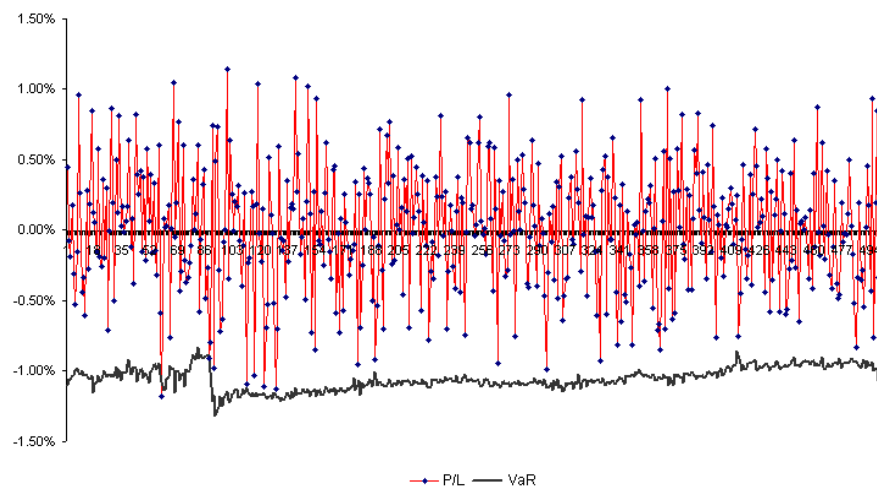


Figure 7: 99% VaR forecasts of Turkish portfolio with MC-DCC

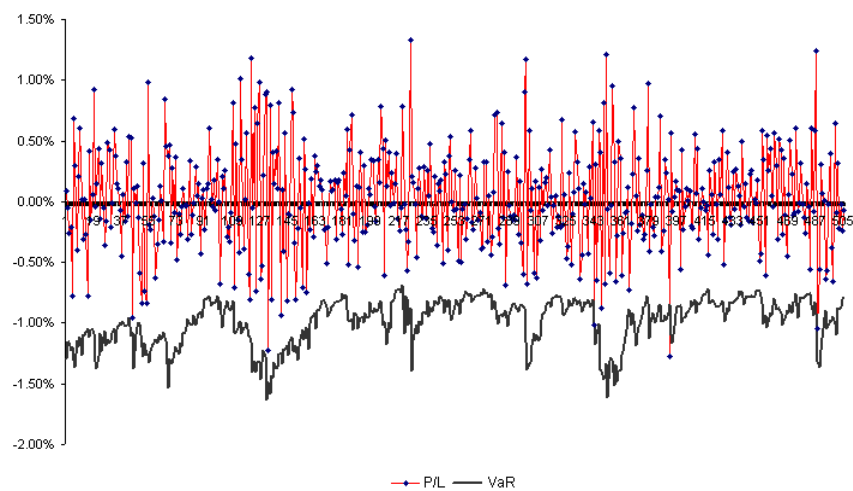


Figure 8: 99% VaR forecasts of US portfolio with HS variants

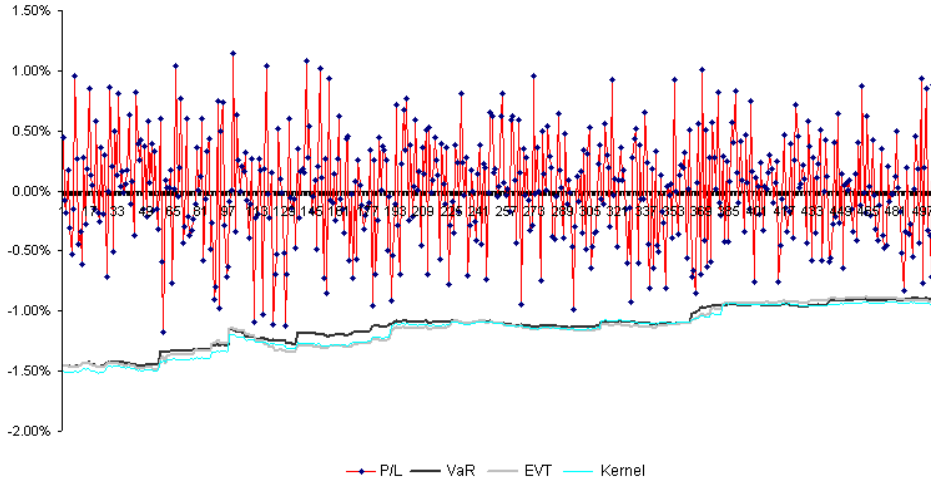


Figure 9: 99% VaR forecasts of US portfolio with FHS-EWMA variants

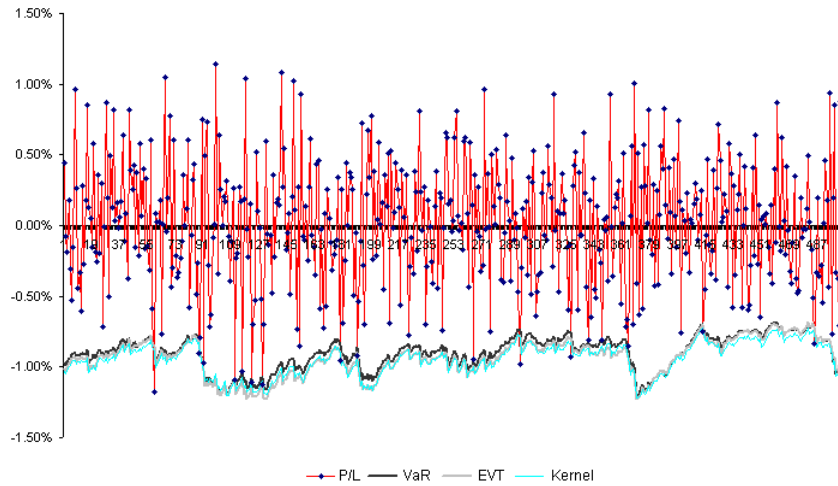


Figure 10:

Figure 11: 99% VaR forecasts of US portfolio with FHS-GARCH variants

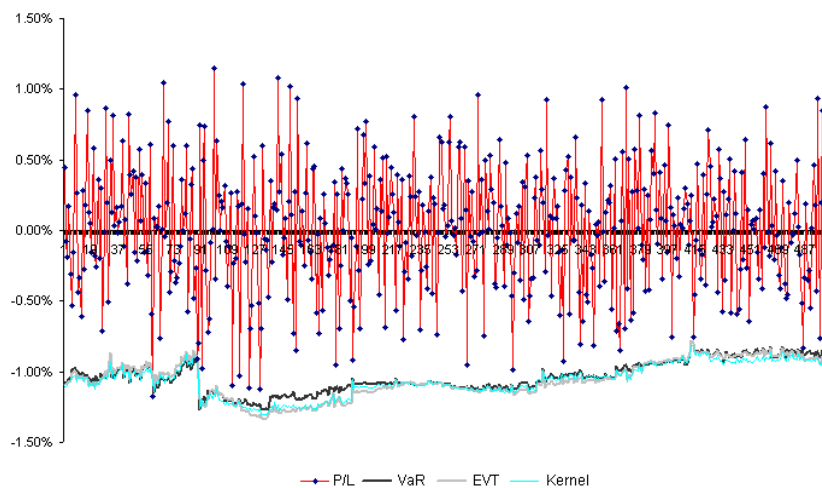


Figure 12: 99% VaR forecasts of US portfolio with WHS

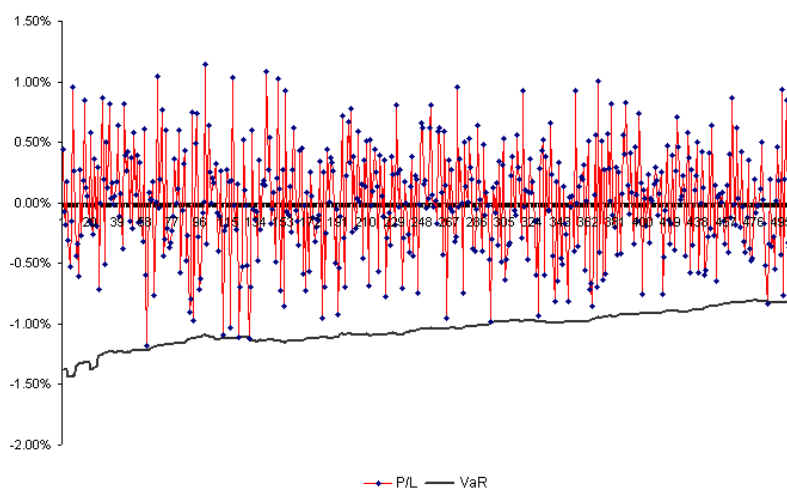


Figure 13: 99% VaR forecasts of US portfolio with MC-EWMA

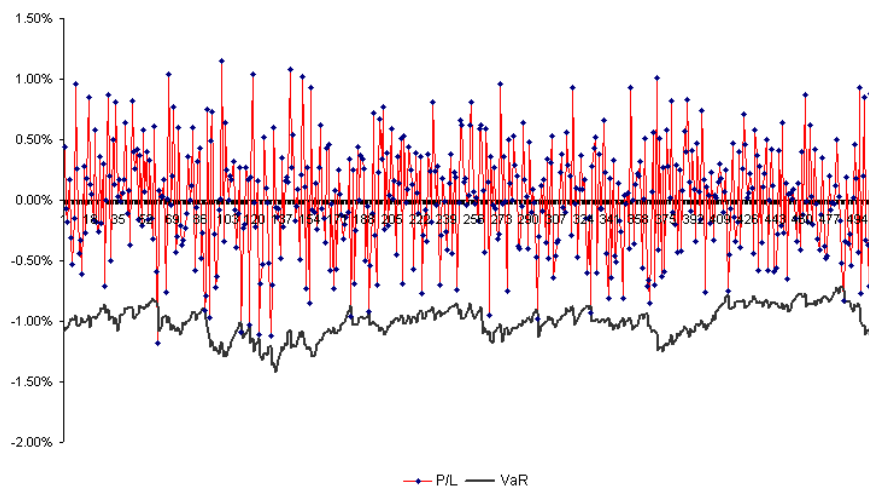


Figure 14: 99% VaR forecasts of US portfolio with MC-CCC

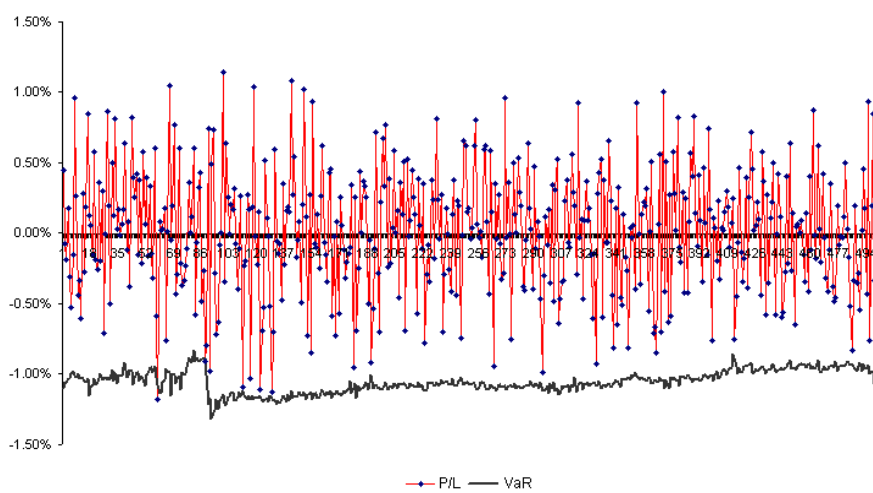


Figure 15: 99% VaR forecasts of US portfolio with MC-DCC

